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Wavelet Spectra of JACEE Events

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Abstract

Pseudo-rapidity distributions of two high multiplicity events Ca-C and Si-AgBr observed by the JACEE are analysed by the wavelet transform. Wavelet spectra of those events are calculated and compared with the simulation calculations. The wavelet spectrum of Ca-C event somewhat resembles to that simulated with the uniform random numbers. That of Si-AgBr event is not reproduced by simulation calculations with Poisson random numbers, uniform random numbers, or a p-model.

Introduction. In high energy ncleus-nucleus (AA) collisions, number density of secondary particles in the rapidity space become very high, and studies of number density fluctuations in the rapidity space is expected to reveal new features of multiparticle production mechanisms. We have analysed pseudo-rapidity distributions in AA collisions including JACEE events, using Higuchi's method (a kind of length method), the auto-regressive model [1], and the fast Fourier transform [2]. In order to classify high multiplicity events by those pseudo-rapidity distribution patterns, the fractal dimensions of the distributions (box dimensions) are estimated with both methods.

In this paper, the pseudo-rapidity distributions of the two JACEE events, Ca-C and Si-AgBr are analysed by the wavelet transform [4],[5],[6],[7]. Any function (data) can be expanded by self-similar wavepackets in this scheme. Therefore characteristics of local fluctuations can be extracted from the data. Wavelet spectra of the two events are calculated and are compared with simulation calculations. In the second paragraph, the wavelet transform is concisely introduced. In the third one, wavelet spectra of the two events are calculated. Those are compare with the simulation calculations in the next paragraph. The final paragraph is devoted to the concluding remarks.

Wavelet transform. Wavelets are constructed from dilation and translation of a scaling function. The scaling function $\phi(x)$ is constructed by an iteration equation,

$$\phi_i(x) = \sum_{k=0}^{N-1} c_k \phi_{i-1}(2x-k) \quad i = 1, 2, \cdots, \tag{1}$$

from a primary scaling function $\phi_0(x)$, where N is a even number, and c_k $(k = 0, 1, \cdots, N-1)$ are constants. The iteration is continued until $\phi_i(x)$ becomes indistinguishable from $\phi_{i-1}(x)$, and $\phi(x)$ is defined by $\phi_i(x)$. The primary scaling function $\phi_0(x)$ is taken as

$$\phi_0(x) = \begin{cases} 1 & \text{for} \quad 0 \le x < 1, \\ 0 & \text{otherwise.} \end{cases}$$

The mother wavelet $W(x)$ is given by

$$W(x) = \sum_{k=0}^{N-1} (-1)^k c_k \, \phi(2x + k - N + 1), \tag{2}$$

and the j-th level wavelet $(j = 0, 1, \cdots)$ is defined as

$$\psi_{j,k}(x) = 2^{\frac{j}{2}} W(2^j x - k), \qquad k = 0, 1, \cdots, 2^j - 1. \tag{3}$$

Coefficients $c_k \, (k = 0, 1, \cdots, N-1)$ should be determined so that the wavelets and the scaling function satisfy the following orthogonal conditions,

$$\begin{aligned} \int \phi(x) \, \phi(x) dx &= 1, \\ \int \phi(x) \, \psi_{j,k}(x) dx &= 0, \\ \int \psi_{j,k}(x) \, \psi_{r,s}(x) dx &= \delta_{jr} \delta_{ks}. \end{aligned} \tag{4}$$

If N=4, the coefficients are given by

$$\begin{aligned} c_0 &= \frac{1+\sqrt{3}}{4}, & c_1 &= \frac{3+\sqrt{3}}{4}, \\ c_2 &= \frac{3-\sqrt{3}}{4}, & c_3 &= \frac{1-\sqrt{3}}{4}. \end{aligned} \tag{5}$$

Arbitrary function $f(x)$ defined in the region $0 \le x < 1$ can be expanded as

$$\begin{aligned} f &= f^{\phi} + \sum_{j=0} f^{(j)}, \\ f^{\phi} &= a_0 \phi(x), \\ f^{(j)} &= \sum_{k=0}^{2^j - 1} \alpha_{j,k} \psi_{j,k}(x). \end{aligned} \tag{6}$$

By the use of Eqs.(4) and (5), We have a relation,

$$\int f(x)^2 dx = a_0^2 + \sum_{j=0} E_j, \tag{7}$$

where E_j denotes the j-th level wavelet spectrum [8],

$$E_j = \sum_{k=0}^{2^j-1} \alpha_{j,k}^2, \qquad j = 0, 1, \cdots. \tag{8}$$

Equation (7) corresponds to the Parseval's equation in the Fourier transform.

Hereafter we consider the case that the function $f(x)$ defined in the region $0 \le x < 1$ is given by descrete value,

$$f_i = f(x_1 + \Delta \cdot (i-1)), \quad i = 1, 2, \cdots, 2^r.$$

We assume that $f(x)$ is periodic outside the defined region. As the data points are 2^r, $f(x)$ is expanded by finite terms. Summation on the right hand side (RHS) of the Eq.(5) runs from $j = 0$ to $j = r-1$.

By the use of column matrices,

$$f = \begin{pmatrix} f_1 \\ f_2 \\ \cdots \\ f_{2^r} \end{pmatrix}, \qquad A_j = \begin{pmatrix} \alpha_{j,0} \\ \alpha_{j,1} \\ \cdots \\ \alpha_{j,2^j-1} \end{pmatrix}, \qquad j = 0, 1, \cdots,$$

Eq.(6) is expressed as

$$\begin{aligned} f &= f^\phi + \sum_{j=0}^{r-1} f^{(j)}, \\ {}^t f^\phi &= 2^{\frac{r}{2}}\, a_0 L_1 L_2 \cdots L_r, \\ {}^t f^{(j-1)} &= 2^{\frac{r}{2}}\, {}^t A_{j-1} H_j L_{j+1} \cdots L_r, \qquad j = 1, 2, \cdots, r-1, \\ {}^t f^{(r-1)} &= 2^{\frac{r}{2}}\, {}^t A_{r-1} H_r, \end{aligned} \tag{9}$$

where L_j and H_j are $2^{j-1} \times 2^j$ matrices. The $i-$th row of L_j has $2(i-1)$'s zeroes from the first to the $2(i-1)$ -th elements, and is expressed as,

$$\begin{aligned} &\begin{pmatrix} 0 & \cdots & 0 & l_{2(i-1)+1} & l_{2(i-1)+2} & \cdots & l_{2(i-1)+N} & 0 & \cdots & 0 \end{pmatrix} \\ &\quad = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & \cdots & 0 & c_0 & c_1 & \cdots & c_{N-1} & 0 & \cdots & 0 \end{pmatrix}. \end{aligned}$$

The $i-$th row of H_j also have $2(i-1)$'s zeroes from the first to $2(i-1)$-th elements and is written as

$$\begin{aligned} &\begin{pmatrix} 0 & \cdots & 0 & h_{2(i-1)+1} & h_{2(i-1)+2} & \cdots & h_{2(i-1)+N} & 0 & \cdots & 0 \end{pmatrix} \\ &\quad = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & \cdots & 0 & c_{N-1} & -c_{N-2} & \cdots & c_0 & 0 & \cdots & 0 \end{pmatrix}. \end{aligned}$$

If $2(i-1)+k > 2^j$ in L_j (or H_j), element $l_{2(i-1)+k}$ (or $h_{2(i-1)+k}$) is added to the $(2(i-1)+k-2^j)$ -th element in each row.

From Eq.(5), matrices L_j and H_j satisfy the following conditions,

$$\begin{aligned} L_j\,{}^tL_j &= I, & L_j\,{}^tH_j &= 0, \\ H_j\,{}^tL_j &= 0, & H_j\,{}^tH_j &= I, \end{aligned} \tag{10}$$

where ${}^tL_j$ denotes the transpose of matrix L_j, and I is the $2^{j-1}-$th order unit matrix. Then, the wavelet coefficients are written as

$$\begin{aligned} a_0 &= 2^{-\frac{r}{2}} L_1 L_2 \cdots L_r f, \\ A_{j-1} &= 2^{-\frac{r}{2}} H_j L_{j+1} \cdots L_r f, \quad j = 1, 2, \cdots, r-1, \\ A_{r-1} &= 2^{-\frac{r}{2}} H_r f. \end{aligned} \tag{11}$$

As the summation on the RHS of Eq.(7) runs from $j=0$ to $j=r-1$ in this case, the wavelet spectra E_j from $j=0$ to $j=r-1$ are obtained.

Wavelet spectra of the data. Pseudo-rapidity η distributions of Ca-C and Si-AgBr events are shown in Fig.1a and b, respectively. From each distribution, we subtract the background distribution [9],

$$\begin{aligned} f_0(\eta) = A\left[(1-e^{-Y-\eta})(1-e^{-y+\eta})\right]^B, \quad -Y \le \eta \le Y, \\ A = 81,\ B = 5.4, Y = 7.0 \qquad \text{for Ca} - \text{C}, \\ A = 184, B = 8.1, Y = 5.5 \qquad \text{for Si} - \text{AgBr}. \end{aligned} \tag{12}$$

Pseudo-rapidty distributions of Ca-C and Si-AgBr events are given with the bin size $\Delta\eta = 0.1$, and the number n of the data points in each event is in the range $2^6 < n < 2^7$. Those used in the analysis should be 2^r. Therefore 64 points ($r=6$) are used for both events; $-3.0 \le \eta < 3.4$ for Ca-C, and $-2.7 \le \eta < 3.7$ for Si-AgBr. Fig 2a and b show the distributions of the two events, where the backgrounds are subtracted. The standard deviation σ of the distribution shown in Fig.2a is 2.71, and that of fig.2b is 4.03.

In Fig.3a and 3b, the wavelet spectra $E_j\,(j = 0, 1, \cdots, 5)$ of Ca-C and Si-AgBr are shown, respectively. The black circles show the wavelet spectra of the distributions where backgrounds are subtracted. The blank circles show that of the raw distributions (without subtraction of the backgrounds). The results are different at the 0-th and the first level, but those from j=2 to j=5 are almost the same. The wavelet spectrum of Ca-C event increases linearly in semi-logarithmic scale from j=2 to j=5. That of Si-AgBr is approximately flat in the same range.

Comparison with simulation calculations. The wavelet spectra of the two events are compared with the following simulation calculations;

(i) Poisson random numbers with a mean μ. The standard deviation σ of the Poisson distribution with the mean μ is given by $\sigma = \sqrt{\mu}$.

(ii) uniform random numbers from 0 to a. The mean μ is given by,

$$\mu = \frac{1}{a}\int_0^a x\,dx = \frac{a}{2}.$$

The variance σ^2 is given by

$$\sigma^2 = \frac{1}{a}\int_0^a (x-\mu)^2\,dx = \frac{a^2}{12}.$$

Then the parameter a is expressed by the standard deviation as

$$a = 2\sqrt{3}\,\sigma.$$

(iii) a p-model[10] with a fraction p_a ($p_b = 1 - p_a$) and an initial 'energy' (or particle number) E_0.

In the p-model, the initial energy E_0 in a rapidity interval Δy is devided into $p_a\,E_0$ and $p_b\,E_0$ at the first step, as the interval Δy splits into two sub-intervals with equal width. $p_a\,E_0$ is regarded as the energy in one interval, and $p_b\,E_0$ is that in the other interval. At the subsequent step, each energy is devided into two in the same way, as each sub-interval splits into two. After n steps, the initial energy E_0 is devided into 2^n terms; energy $p_a^j p_b^{n-j} E_0 \quad (j = 0, 1, \cdots, n)$ appears ${}_nC_j$ times. Those are energies in 2^n ordered sub-intervals of Δy with the same width. The mean μ and the variance σ^2 are given respectively by

$$\begin{aligned}
\mu &= \frac{1}{2^n}\sum_{j=0}^{n} E_0\,p_a^j\,q_b^{n-j} = \frac{E_0}{2^n}, \\
\sigma^2 &= \frac{1}{2^n}\sum_{j=0}^{n}\left(E_0\,p_a^j\,q_a^{n-j}\right)^2 {}_nC_j - \mu^2 = \left(\frac{E_0}{2^n}\right)^2\left(2^n\left(p_a^2+q_a^2\right)^n - 1\right).
\end{aligned}$$

In the simulation calculations, we iterate $r + 2 = 8$ steps, and randomize the ordering of the energies. Then, we add every four energies, and have 2^6 ones, which are compared with the data.

Each simulation calculations are done 1000 events. The mean value is not sensitive to the wavelet spectra, and parameters are adjusted to reproduce the standard deviation of each distribution shown in Fig.2a or b.

We calculate the wavelet spectra $E_j^{simu} \quad (j = 0, 1, \cdots, 5)$ of simulated events and count the number of events within a value of z,

$$z = \sqrt{\sum_{j=2}^{5}(\log_2 E_j - \log_2 E_j^{simu})^2}. \tag{13}$$

In order to generate the random numbers, we use a congruence method[11]. At first, we choose an integer r_0, using the equation

$$r_j = a\, r_{j-1} + c \qquad \mathrm{mod}\ m, \qquad j = 1, 2, \cdots, \tag{14}$$
$$a = 1229, \quad c = 351750, \quad m = 1664501,$$

we generate the random numbers r_j $(j = 1, 2, \cdots)$ subsequently. From those random numbers we get the uniform random number

$$s_j = \frac{r_j}{m}, \quad 0 \leq s_j < 1.$$

The results for Ca-C and Si-AgBr events are shown in Table 1a and 1b, respectively. Table 1a shows that the wavelet spectrum of simulated events for Ca-C events have more than 15 % out of 1000 simulated events within $z < 0.8$ in all the three cases. Those with uniform random numbers have 20 % and is higher than the other two cases.

Contrary to the Ca-C event, simulated events for Si-AgBr event have scarecely low rate within $z < 0.8$ in all the three cases (See table 1b). The wavelet spectrum of Si-AgBr event cannot be reproduced by any of these simulation calculations.

Examples of the simulation calculations with (i) Poisson random numbers are shown in Fig.4a and b, with (ii) uniform random numbers in Fig.5a and b, and with (iii) the p-model in Fig 6a and b.

Concluding Remarks. Two high multiplicity events Ca-C and Si-AgBr are analysed by the wavelet transform. Wavelet spectra of the two events are calculated and compared with the simulation calculations. Wavelet spectrum of the Ca-C event somewhat resembles to the simulation calculations with uniform random numbers, but that of the Si-AgBr event can not be reproduced by any of the three simulation calculations. Further investigation will be reported elsewhere that there are any simulation calculations which reproduce η distributins of high multiplicity events. Our analysis show that observed high multplicity events can be classified by the wavelet spectra, and what statistical regularity (or irregularity) would be dominant event by event.

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Table Caption

Table 1 Number of simulated events which satisfy the condition $z < 0.4$, $z < 0.6$, or $z < 0.8$ for a) Ca-C and b) Si-AgBr. Each simulation calculations are done 1000 events.

Figure Captions

Fig.1 Pseudo-rapidity distributions of a) Ca-C and b) Si-AgBr events.

Fig.2 Distributions used for wavelet analysis. Those are obtained from η distributions where the backgrounds are subtracted; a) Ca-C, and b) Si-AgBr.

Fig.3 Wavelet spectra of a) Ca-C and b) Si-AgBr events. Black circles are calculated from the η distributions where the backgrounds are subtracted. Blanck circles are from η distributions (where backgrounds are not subtracted).

Fig.4 Wavelet spectra of simulated events with Poisson random numbers for a) Ca-C with $r_0 = 44255$ (z=0.14) and b) Si-AgBr with $r_0 = 115383$ (z=0.50).

Fig.5 Wavelet spectra of simulated events with uniform random numbers for a) Ca-C $r_0 = 6634423$ (z=0.25) and b) Si-AgBr with $r_0 = 115383$ (z=0.55).

Fig.6 Wavelet spectra of simulated events with the p-model for a) Ca-C with $r_0 = 1567321$ (z=0.27) and b) Si-AgBr with $r_0 = 707048$ (z=0.58).

criterion	Poisson ($\mu = 7.6$)	Uniform ($a = 9.40$)	p-model ($p_a = 0.36, E_0 = 380$)
$z < 0.4$	18	25	18
$z < 0.6$	61	75	65
$z < 0.8$	152	197	168

a

criterion	Poisson ($\mu = 17.0$)	Uniform ($a = 14.8$)	p-model ($p_a = 0.36, E_0 = 590$)
$z < 0.4$	0	0	0
$z < 0.6$	2	2	2
$z < 0.8$	7	8	10

b

Table 1

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